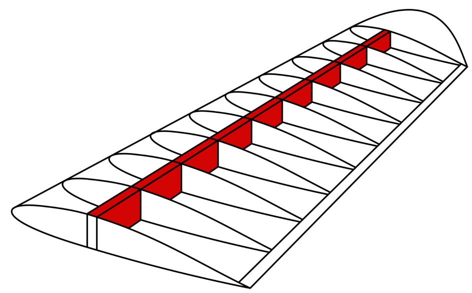
AEROSP 215 **Open-Ended Problem 2** **Written part due 4/12/19 at 8:30 am**
Winter 2019 **Mandatory in-class discussion on 4/12/19**

Possible points: 50

In Open-Ended Problem 1 you focused on the statics of the entire *Nomad* aircraft. Now, in Open-Ended Problem 2, you'll focus on the mechanics (stress and deflection) of one particular part of the *Nomad* – the main spar of one wing. If you don't know what a spar is, it's the main structural component of the wing that runs down the length of the wing. An example is shown in red in the figure to the right. In Open-Ended Problem 2 we will analyze the spar of *one wing* as a flexible beam with an elastic modulus E, a constant moment of inertia I (meaning the beam is prismatic), and a welded connection to the fuselage at $x = 0$.



We will also assume that the beam is loaded by a distributed load $w(x)$ described by the function:

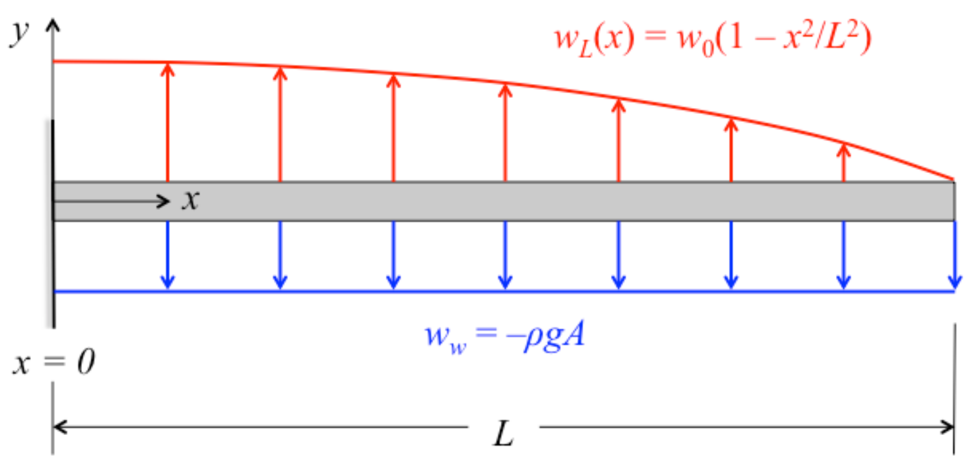
$$w(x) = w_L(x) + w_w = w_0\left(1 - \frac{x^2}{L^2}\right) - \rho g A$$

In this function there are two components to the distributed load.

1. A lift force $w_L(x)$ that varies with x and is described by the equation $w_L(x) = w_0\left(1 - \frac{x^2}{L^2}\right)$ where w_0 is a constant and L is the length of the wing.

2. A weight force w_w that is constant with x and is described by the equation $w_w = -\rho g A$ where ρ is the density of the beam material, g is the acceleration due to gravity (32.2 ft/s^2), and A is the cross-sectional area of the wing. Note that in this model we are only considering the weight of the beam, not the weight of the wing ribs or skin.

Note that the lift points up ($+x$) and the weight points down ($-x$), as you would expect. These distributed loads can be seen in the figure below:



As with Open-Ended Problem 1, there are two parts to this assignment:

- A written part that is due to Gradescope by Friday, April 12 at 8:30 am. This part is worth 40 points, broken down as such:

Part	**1**	**2**	**3**	**4**	**5**	**6**
Points	5	8	15	6	3	3

- Participating in a small-group discussion during class on Friday, April 12. You should bring a copy of your written work to this discussion, as you'll be comparing your work with a small group of other students. This part is worth 10 points.

First, you will do some "closed-ended" calculations with this distributed load to set you up for the open-ended part. Your answers for Parts 1 and 2 should be symbolic expressions in terms of some set of known variables w_0, L, ρ, g, A, E, and I. As these parts have correct answers, we have provided an answer checker on Canvas.

Part 1. Draw an FBD of the spar in one wing and calculate the reactions at the point where the spar is welded to the fuselage (at $x = 0$).

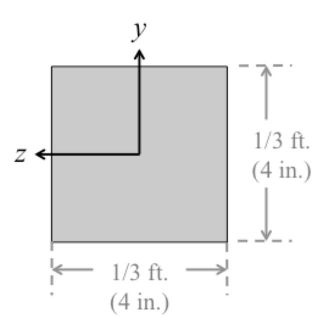
Part 2. Write an equation for the bending moment $M(x)$ throughout the beam.

Now let's plug in some numbers. Looking at the poster beside the Nomad we see that $L = 9.75$ ft, and we have also estimated that $w_0 = 168$ lb/ft based on the Nomad's specs.[a] What about E, I, ρ and A? The ρ and E of the spar beam are functions of the beam material, and I and A are functions of the beam geometry. So, the choice of a material and geometry influence the beam's weight *and* the stress experienced by the beam *AND* the safety factor—which is also a function of your chosen material's yield stress!

And, as we discussed in Open-Ended Problem 1, strength isn't the only design parameter used in real-world engineering. **Cost** is another important one. We want a spar beam that's strong enough for our application but that doesn't break the bank. In other words, we're asking:

What's the lowest-cost spar beam that has an aerospace-appropriate safety factor of 1.5?

In the rest of this problem you're going to explore these interdependencies and answer this question. While it's not required, we *strongly* encourage you to use software to help you do Parts 3-5. This could be MATLAB, Excel, or another program. Using software will help you to automate the process and you won't have to write the same equation again and again. You should also submit a print-out of the code that you write with your work.



Limits of Beam Cross-Section

Lastly, we'll put some limits on the maximum dimensions of the beam geometry. Measuring the *Nomad*, it looks like the spar beam cross-sectional geometry should fit within an area 1/3 ft. by 1/3 ft. Also, the minimum size of any given dimension (b or h in a rectangular beam; a, b, g, or h in an I- or T-beam) should be 1/48 ft.

Part 3. As we mentioned above, the two main design decisions are the beam material and geometry. And the geometry is itself a function of a number of parameters of the cross-sectional shape—rectangular, I-beam, T-beam, or another shape that we haven't discussed in class.[b] In this part, we want you to do a **parameter sweep**. Choose one parameter to vary (with at least 10 different values) and hold the rest constant. For example, maybe you decide to look at a rectangular beam made of spruce wood. Holding the material and height constant, you vary the width.

For each configuration, calculate the beam safety factor and cost. Make two plots:

- The safety factor (y-axis) vs. the parameter you're varying (x-axis)
- The cost (y-axis) vs. the parameter you're varying (x-axis)

Describe the effect that varying the parameter has on the beam safety factor and cost.

Part 4. Find a mathematically-feasible combination of material and geometry that gives a safety factor of at least 1.5. (Note: this combination *does not* have to be part of your parameter sweep in Part 3.) Clearly describe your chosen material and geometry. With your chosen beam material and cross-sectional geometry, calculate:

- The maximum deflection of the beam and the location where it occurs.
- The total cost of the beam (all 9.75 ft. of it).

You need to write or type these answers in your submitted work (i.e. we're not going to hunt for them in your code) *and* you need to enter your material, beam geometry, maximum deflection, and total cost in this Google Form. If you do not enter your numbers in this form you will receive -3 points on Part 4.

The question we posed above was "*What's the* lowest-cost *spar beam that has an aerospace-appropriate safety factor of 1.5?*" which is the perfect opportunity for a class-wide contest! I will award bonus points to the seven lowest-cost beams that have a safety factor ≥ 1.5. (There is no benefit to having a safety factor above 1.5.) The lowest-cost beam will receive 7 bonus points, the second-lowest-cost beam will receive 6 bonus points, and so on until the seventh-lowest-cost beam will receive 1 bonus point. (A reminder about the honor code here: I encourage you to discuss your method for optimizing the beam material and cross-sectional geometry with your peers, but you should not just let your peers copy down your answer. Doing so would be a violation of the honor code.)

Now, finish this problem by thinking about how this problem translates to the real world:

Part 5. In Part 4 your beam geometry only had to be mathematically feasible. But do you think it's *physically* feasible? Could Ed Lesher have really built the spar bam to these specifications? Justify your answer.

Part 6. Briefly describe a way that you could change this model of the *Nomad*'s wing and the loads on it to be more accurate (and therefore more complex). With these changes, could you still solve for the bending moment, stress, and deflection?

Material Properties Table

Material	Cost ($/lb)	Density (ρ, slug/ft^3)	Elastic Modulus (E, 10^6 psi)	Yield Stress* (σ_{yield} , 10^3 psi)
Aluminum alloy (7075-T6)	0.82	5.24	10.15	72.52
Beryllium alloy	142.88	5.63	35.53	52.21
Brass (70Cu30Zn, annealed)	1.00	16.30	18.85	10.88
Concrete	0.02	4.85	6.96	0.44
Copper alloys	1.02	16.10	19.58	73.97
Granite	1.43	5.04	9.57	8.70
Lead alloys	0.54	21.53	2.32	4.79
Nickel alloys	2.77	16.49	26.11	130.53
Spruce (parallel to grain)	0.45	1.16	1.31	6.96
Steel, mild 1020	0.23	15.13	30.46	29.01
Steel, stainless austenitic 304	1.22	15.13	30.46	34.81
Titanium alloy (6Al4V)	7.37	8.73	14.50	131.98

* Assume that the yield stress is the same in tension and compression.
Note: The values for concrete and granite are actually failure stresses, as these materials are ceramics that break before they yield.

Data from https://ocw.mit.edu/courses/materials-science-and-engineering/3-91-mechanical-behavior-of-plastics-spring-2007/readings/props.csv

[a] In case you were wondering, here's how we estimated w_0. We used the standard lift equation from Aero 201: $L = \frac{1}{2}C_L r v^2 A$. If you divide each side by the wingspan, you get the distributed lift in units of [force/length]. So, we have the equation $w_0 = \frac{1}{2}C_L r v^2 c$ where the lift coefficient $C_L = 0.938$ (the maximum for a NACA 23015 airfoil according to this website), the air density $r = 2.31\times10^{-3}$ slugs/ft^3 (at an altitude of 1,000 ft), the speed v = 120 mph = 176 ft/s (the normal cruise speed of the *Nomad*), and the chord c = 40 in = 3.33 ft.

[b] If you choose a cross-sectional shape that hasn't been discussed in class, you need to make sure it's symmetric about the y-axis.